Helping Students With Mathematical Disabilities to Succeed

Elizabeth Wadlington and Patrick L. Wadlington

ABSTRACT: Teachers and parents are often perplexed when an intelligent student performs poorly in mathematics. Research tells us that this is often due to math disability, otherwise known as *dyscalculia*. The authors define dyscalculia and describe its major subtypes. Also, the authors describe characteristics of dyscalculia and explain why dyscalculia is difficult to assess and identify. In addition, the authors explore language difficulties, math anxiety, and other factors that may cause poor mathematical performance. They also discuss quantitative and qualitative mathematical learning styles and their effects on math performance. Last, the authors explain proven successful intervention strategies that teachers can use to help students with dyscalculia to succeed in mathematics.

KEYWORDS: *dyscalculia, learning disabilities, math disability, math learning problems*

> KATRINA IS 10 YEARS OLD and in the fourth grade. She is a lively, intelligent student who excels in oral presentations. Her grades in reading and writing are above average, and she loves to read biographies and historical novels. Recently, she won first place in her school's social studies fair. However, she did not place in the science fair because the mathematical calculations in her project were incorrect.
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> Katrina, in fact, is failing in her math class this year. Her teacher has warned Katrina that if she does not pass the mathematics component of the statewide test, she will not be able to advance to the fifth grade. Katrina's parents have provided her with a math tutor for the last 3 years, but Katrina has still been unable to master basic facts. Although a calculator has somewhat helped Katrina circumvent this problem, she still has difficulty in understanding and applying concepts such as fractions, percentages, shapes, and measurements. Actually, Katrina has a problem remembering any sequence of numerals. Last year, she was left behind on a school trip and became hysterical when she could not remember her telephone number to call her parents.

Like Katrina, many people have problems with mathematics. Because these people often have great strengths in other areas, their mathematical difficulties tend to be unexpected or ignored. Specific mathematical difficulties are diverse; therefore, addressing each individual's problems can be a challenge for students and their teachers. In addition, because most students with mathematical difficulties learn in a mainstream educational classroom, many teachers do not have the specialized expertise needed to help them. In this article, we try to help students and teachers by answering the following questions: (a) What is a math disability? (b) What are its characteristics? (c) What can teachers do to help students with a math disability?

Definition and Prevalence

Severe problems with mathematics are referred to as *dyscalculia*. Although exact definitions vary, it is generally agreed that dyscalculia is a neurologically based disorder of mathematical abilities. It originates as a genetic or congenital disorder of the brain and causes a discrepancy between an individual's general cognitive level and mathematical abilities (Geary, 2000; Kosc, 1974; Rourke & Conway, 1997; Sharma, 1990; Weedon, 1992). In the present article, we use the terms *mathematical disability* and *dyscalculia* interchangeably.

It is difficult to determine the prevalence of mathematical disabilities because of differing definitions and the overlap of diverse learning disabilities. In addition, there is no universal set of criteria for diagnosis (Geary, 2000). However, research indicates that at least 6% of the school-aged popu-

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lation has dyscalculia (Badian, 1983, 1999; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974). Specifically, Badian (1999) reported a prevalence rate of 6.9%. Of these same students, 3.9% scored low in arithmetic only and 3% scored low in both arithmetic and reading. Geary (1993) suggested that mathematical disorders are as common as reading disorders and that the same deficit may cause mathematical and reading disorders to simultaneously occur in some individuals.

Dyscalculia has been divided into subtypes in various ways. Geary (2000) developed the following useful categorization:

1. *Semantic memory:* difficulty retrieving arithmetic facts.
2. *Procedural memory:* difficulty understanding and applying mathematical procedures.
3. *Visuospatial memory*: difficulty understanding spatially represented numerical information such as misalignment of columns, place value errors, or geometry.

The fact that individuals with dyscalculia have differing subtypes of mathematical disability makes diagnosis and treatment perplexing. It is also confusing because a learning disability may not be the only cause of mathematical difficulties.

Other Causes of Mathematical Difficulties

Language Difficulties

Mathematics has its own language that comprises symbols (e.g., %, $, =) and special terms (e.g., *numerator, percent, addend*). Many terms (e.g., *denominator, prime, factor*) have one definition in mathematics and another definition in other subjects. Language processing disabilities such as dyslexia (i.e., reading disability) or dysgraphia (i.e., writing disability) can hinder a person's ability to learn vocabulary and concepts and use symbols, signs, and operations. For example, people with language-processing disabilities often have problems with directionality, sequencing, and organization. However, mathematics requires individuals to work in a specific direction, follow steps correctly, and work in an organized way. For poor readers, decoding and comprehending mathematical instructions, texts, and word problems can be challenging. In addition, written mathematical work can be hard for people with difficulties in handwriting, spelling, or writing composition.

Furthermore, individuals who have auditory comprehension problems often have difficulty learning concepts orally, whereas those with oral language or word-retrieval deficits have difficulty explaining concepts aloud. People who inefficiently process any of these aspects of language will often not be able to do large amounts of mathematical work in a timely manner (Alvermann, Swafford, & Montero, 2004; Farmer, Riddick, & Sterling, 2002; Henderson, 2001; Kintsch, 2004; Marolda & Davidson, 2000; Tomey, Steeves, & Gilman, 2003).

Math Anxiety

Even young children know who is and is not good at mathematics. Many students, with or without mathematical disabilities, who perceive themselves as not good in mathematics develop a fearful avoidance of mathematical situations known as *math anxiety*. It can range from mild to severe. Math anxiety is not a learning disability. However, it interferes with an individual's ability to learn mathematics and perform on tests. A cycle develops in which failure in mathematics leads to anxiety, which leads to more failure, which leads to more anxiety (Farmer et al., 2002; Kogelman & Warren, 1978).

According to Sharma (1990), math anxiety may be specific or global. In *specific math anxiety*, an individual is anxious about a particular mathematical situation. This person is usually not afraid of mathematics in general and believes that he or she can learn it with appropriate instruction. In *global math anxiety*, an individual feels stressed in all mathematical situations and abhors all aspects of mathematics, and he or she spends time and energy in avoiding mathematics.

Additional Factors

Other factors make it difficult to determine whether students' mathematical difficulties are due to dyscalculia or other problems. For example, lack of motivation can affect students' abilities to learn mathematics (Farmer et al., 2002; Spafford & Grosser, 1996). Also, a teacher's lack of experience and confidence can lead to poor instruction, which prevents learning (Marshall, 2003). In addition, a student's lack of prerequisite skills can hinder the learning of new information (Levine, 1993; Spafford & Grosser). Less attention has been focused on dyscalculia than on language disabilities; therefore, dyscalculia is less likely to be differentiated from other causes and treated appropriately (Badian, 1999; Farmer et al., 2002; Spafford & Grosser).

Mathematical Learning Styles

Although individuals may understand sophisticated mathematical concepts well, they may be unable to perform basic mathematical operations or may fail to read signs or symbols correctly when they apply these concepts. Students who do not understand basic mathematical concepts try to memorize procedures without understanding them; this attempt results in an inappropriate use of procedures. Unfortunately, some people have both types of problems (Sharma, 1990). According to Sharma, there are two mathematical learning styles or personalities that teachers should consider when planning instruction: qualitative and quantitative.

Quantitative Personality or Learning Style

Individuals with quantitative personalities are usually good with language skills and concepts. They are sequentially oriented and like to take apart problems, solve each piece, and then reassemble those pieces. They quantify information well and solve word problems in an organized, sequenced, and logical manner. They are more part-to-whole oriented and may have difficulty perceiving holistic relationships (Sharma, 1990).

People who approach mathematics quantitatively are usually good at skills such as calculation procedures, counting, addition and multiplication, fractions interpreted by verbal means, and naming geometric shapes. They usually have difficulty with topics such as broad concepts and principles, estimation, selection of operations in word problems, fraction concepts, sophisticated geometry, and the use of flexible approaches (Marolda & Davidson, 2000).

Qualitative Personality or Learning Style

Individuals with qualitative personalities focus on the visual–spatial, holistic, and inductive aspects of mathematics and use intuitive approaches. They learn mathematics best by perceiving patterns and relations. They skip steps and develop new techniques for solving problems. Individuals with quantitative personalities are more part-to-whole oriented, whereas individuals with qualitative personalities are more whole-to-part oriented, and exhibit problems with sequence and detail. Consequently, they frequently make careless errors and lack automaticity in the basics. Their written work is often sloppy. Geometry is usually easier for them to learn than are other mathematical concepts (Sharma, 1990).

People who approach mathematics qualitatively are usually good at concepts such as backward counting, subtraction and division, estimation, fractions interpreted by visual models, and spatial relations between and manipulation of geometric shapes. They are usually challenged by tasks such as noting details of multistep procedures or of word problems, using multistep calculations, finding and appreciating exact solutions, performing formal fraction operations despite understanding concepts, using exact mathematical language, and using a single, specified approach (Marolda & Davidson, 2000).

People with quantitative personalities learn best through direct, sequential, step-by-step teaching methods. Those with qualitative personalities learn best through inductive, visual–spatial, and pattern strategies. Although one type of mathematical personality usually remains dominant, doing well in mathematics requires integrating quantitative and qualitative approaches. That is, mathematics teachers must provide instruction that includes deductive–inductive strategies, sequential step-by-step procedures, holistic–visual–spatial strategies, and pattern recognition (Sharma, 1990). When individuals understand their own learning styles and how to adjust them to fit diverse situations, they gain a sense of control over their own learning (Farmer et al., 2002).

Intervention Strategies

Interventions should be based on the Curriculum and Evaluation Standards of the National Council of Teachers of Mathematics (NCTM; 2000). The NCTM states that all students can learn mathematics; however, *equity* is not synonymous with treating students the same. All students need appropriately high expectations, a challenging curriculum, and effective instruction.

Teachers have found the following interventions useful when they work with students with dyscalculia and with mathematical difficulties due to other causes. Many of these strategies are examples of good teaching for everyone, but they are imperative for students with problems in mathematics. These interventions should be used as a springboard to refine strategies for individual students in their own unique situations. These suggestions are addressed to teachers, but tutors, parents, and even students may adapt them.

Treatment of Math Anxiety

Often before students can be successful in mathematics, they must start to overcome math anxiety. Conversely, becoming successful in mathematics will help students to overcome math anxiety. Initially, teachers should assure their students that the classroom environment is safe. Punishment or ridicule from the teacher or other students for incorrect answers is always inappropriate. Teachers should assess students in nonthreatening ways and provide activities that produce mathematical success. Charts and graphs can be used as visuals to help students monitor their own progress. Students' names and daily activities can be used in word problems to motivate them and help them see reasons to use mathematics.

Teachers can help each student to become an expert in some aspect of mathematics. Perhaps one student can become good at illustrating word problems; another can become skillful in choosing the operation to solve problems; still another can excel at lining up vertical problems. Teachers should praise students frequently for large and small successes. They should encourage their students to value progress in learning rather than value only correct answers.

Furthermore, teachers can share personal anxiety-producing situations and tell their students how they overcame anxiety. Teachers can also discuss famous people (e.g., Einstein, Churchill) who overcame learning difficulties. Then, teachers can encourage their students to make a plan to alleviate their anxiety. Steps for overcoming anxiety (e.g., take deep breaths, believe in oneself, get help) can be discussed and posted.

Math games can foster good attitudes about mathematics and can provide opportunities for social interaction and success in the subject. By allowing students to work in pairs to do mathematical tasks together, teachers can also encourage positive attitudes and enhance communication skills in mathematics. Teachers should always encourage cooperation rather than competition between students. One good way to encourage cooperation is to form learning groups.

General Instruction

Students with mathematical difficulties should be seated near the focus of instruction and should actively engage in lessons. Instruction should be well organized with new ideas logically building on old ones. Teachers should preview the lesson and its objectives before they begin instruction. For students to construct their own knowledge in meaningful ways, it is best for teachers to start instruction with concrete objects and then move to pictures and diagrams. They should present students with abstractions only after multiple direct experiences with concepts. Before moving on, students should master prerequisite skills. If this is impossible, teachers should teach them how to circumvent the problem that the lack of the prerequisite created (e.g., use a calculator for long division, type answers to problems if handwriting is illegible).

After reviewing the big picture, teachers should break down skills and concepts into their smallest parts and present them step by step. Teachers should model new ideas and provide ample time for guided practice before having students work on their own. At the end of each mathematics class, students should summarize what they have learned. At the beginning of the next mathematics class, students should again explain and demonstrate what they learned during the previous class. Frequent short reviews are more conducive to learning than one long review immediately before an exam. Teachers should keep in mind that time pressures and large amounts of work overwhelm students with difficulties in mathematics. Therefore, teachers should break down long assignments into smaller ones and provide students with immediate feedback after each one.

It is important that textbooks and other materials are appropriate. Textbooks should convey the major concepts thoroughly rather than cover many small ideas briefly. New skills might be presented in isolation, but they should then be integrated into other areas and applied to real life. Materials should be on the appropriate ability and interest levels. That is, sometimes teachers will have to duplicate materials from easier texts (being sure to remove grade-level labels to avoid discouragement) or create materials of their own.

Students learn best through multisensory instruction that incorporates multiple senses and movement (Clements, 2000; Marolda & Davidson, 2000; Spafford & Grosser, 1996; Tomey et al., 2003). Manipulatives such as geoboards, Base 10 Blocks, Cuisenaire rods, geometric blocks, counters, and play money make abstract concepts concrete. Using measuring tools to measure real objects, walking a number line to add or subtract, using real Celsius and Fahrenheit thermometers to measure and convert temperature, and filling a jar with jelly beans to illustrate numbers are examples of multisensory activities that are much more effective than are worksheet drills. Tracing the numerals or words in the text while reading aloud or subvocalizing is often helpful.

Teachers should allow extra time so that students can overlearn mathematical skills until these skills become automatic. Although understanding should always be emphasized, automaticity requires memorization. If possible, students should memorize basic facts. Teachers should break memorization tasks into chunks (e.g., first half of the +6s, then second half of the +6s) so that students do not become frustrated with how much they must memorize. Students should learn through multisensory strategies such as saying facts as they write them or trace them, practicing facts aloud in unison with others, or playing math games. Teachers should help them to see the relations between facts (e.g., $2 + 7 = 7 + 2$) and brainstorm memory tricks. Often putting facts to music or rhythm facilitates learning (Beal, 2000; Edelson & Johnson, 2003; Fernandez, 1999; Powell, 1998; Rusin, 2004). Speed should not be emphasized until facts are mastered. Then, students should be encouraged to time themselves to discourage stress. If students take a great deal of time to memorize facts, teachers should allow them to use calculators, fact charts, computers, and counters (while they continue to learn facts) so they can proceed to higher mathematical learning.

Teachers should prioritize objectives and emphasize practical skills and concepts appropriate for students' life stages. Younger children need to master tasks (e.g., telling time so they will not be late for school, being able to count money for lunch, using a calendar to remember to turn in assignments on time). As students become older, they need to progress to learning other tasks (e.g., how to write a check, balance a checking account, interpret basic statistics in a newspaper, figure percentages and interest on loans).

Students profit from extra instruction with a math tutor. Math tutors should understand child development and learning theory, be well grounded in mathematics, and know the best ways to work with students with learning disabilities. They also must be able to explain mathematics in understandable language with real-world examples. In addition, they should develop good rapport with students and help them to build mathematical confidence.

Mathematical Communication

Students should have opportunities to communicate mathematics in multiple ways. Writing in a mathematics journal helps many students to become comfortable with mathematical terms and ideas. Student may write about diverse aspects of mathematics (e.g., attitudes, successes, difficulties, thought processes in figuring out a problem, applications to real life) in their journal entries. Initially explaining a procedure or a concept in writing can prepare students to discuss their ideas orally. In contrast, some students may need to discuss mathematical ideas aloud before they attempt to write about them. Teachers should not lower scores because of misspellings in writing of this nature.

Teachers should explain and model new vocabulary using concrete examples; then, they can ask students to explain and give their own examples. Students can create their own mathematical dictionaries to illustrate new terms. Using their dictionaries as resources, they can play definition-guessing games and then exchange dictionaries with each other to play again. Using multiple examples, teachers should teach new signs and symbols. Students can discuss how symbols are alike or different and can create their own memory tricks to remember them (e.g., the big side of the > symbol faces the larger number). Teachers should stress meaning rather than rote memorization. Students should have informal opportunities to talk about mathematical ideas using correct vocabulary (e.g., how much money is needed to take a field trip, how many days of school are left).

Teachers should avoid unnecessary paralanguage (e.g., "Wait to sharpen your pencil"; "Please sit up straight") when they give directions or explain ideas. Some students may need to tape-record directions or explanations so that they can listen again later. It is also helpful to post directions and procedures for future reference. Teachers can pass out reminder sheets with important information written in accessible language for students to keep in their notebooks or taped to their desks.

Students who have poor reading abilities may need help in comprehending a mathematics textbook. Books can be tape-recorded, or someone else can read them aloud. As students listen to the text, they should highlight keywords. If students are not allowed to write in textbooks, they can use sticky notes with arrows for this purpose. In their own words, they should immediately paraphrase the reading task.

Students with poor handwriting may also need special assistance. Numerals and symbols should be posted and included with the alphabet for handwriting practice. Students can write on graph paper to align numerals more easily. Students can put their papers under the problems in the text or teachers can provide duplicate copies of problems; consequently, students could just record answers.

Teachers and students can find satisfaction in work that is legible, neat, and organized, even when the handwriting is far from perfect. The time that is spent on recopying work that students have already mastered is better spent on new learning. In some instances, students may need to use a word processing program to make their math work legible.

Assessment

Ongoing formal (e.g., tests) and informal (e.g., observation, checklists, interviews) assessment procedures are necessary to plan appropriate instruction. Important decisions (e.g., placement, remediation) should involve multiple kinds of assessment, and decisions should be reviewed periodically. Assessment should always take place in a relaxed, stress-free environment.

Written tests should be legible with good spacing. Items should be grouped according to type of format (e.g., vertical problems, horizontal problems, multiple-choice items, matching items). It may be necessary to reduce the number of items on a test or break the test into parts with more than one testing session. Students may need to write directly on the test instead of an answer sheet. Sometimes students may need to have parts of the test read aloud.

Interviews are another way to gain information to plan instruction later. Teachers can ask students to assess their own mathematical strengths and weaknesses. Students can explain strategies that seem to help. They can work on problems as the teacher observes and can demonstrate concepts by using manipulatives. Teachers may use checklists and rubrics to record interview observations. Older students can use these same instruments to track their progress.

Analysis of error patterns in computation is another important assessment tool. Appropriate classifications will depend on the students' abilities, but common error patterns include but are not limited to the following: (a) basic facts error, (b) wrong operation, (c) sequencing error, (d) misalignment of numerals, (e) regrouping error, (f) directionality error, (g) copying mistake, (h) reading error, and (i) random error. It is important to catch errors quickly before students begin to practice them.

When informal screening indicates that a student may have a mathematical disability, more in-depth diagnosis by a team of experts (e.g., teachers, mathematicians, psychologists, parents, self) is warranted. Rather than focus on a label for the student's difficulties, the team should concentrate on specific areas of difficulty around which to plan interventions.

Word Problems

Mathematical word problems are more motivating when they relate to students' experiences. As in real life, there should be more than one way to solve a problem, and there might be more than one right answer. Students often benefit from writing their own word problems and exchanging them with each other.

If poor reading skills hinder students, then the teacher should read the problems aloud and discuss them before students attempt to solve the problems. Also, students should code word problems (e.g., underline the facts, blacken the distracters, circle the questions). After they are sure they understand the problem, students should devise a plan for solving it. To do this, they need a toolbox of heuristic strategies (e.g., draw a picture, work backwards, practice visual imagery, use manipulatives). Once the students have solved the problem, they should ask themselves, "Does this answer make sense?" Then, they should check their work.

Conclusion

Dyscalculia and other mathematical difficulties can be perplexing to teachers and students. As inclusion of students who have serious difficulties in mathematics becomes more prevalent, knowing how to work with students' problems will become more and more important to all classroom teachers. Interventions such as the aforementioned ones should be a place to begin: Teachers and students will undoubtedly discover others. In their doing so, schools will move closer to their goals of educating all students and leaving no children behind.

NOTE

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